TOPOLOGY (MATH 54)
TAKE-HOME MIDTERM EXAM

NAME: ____

Instructions: This is an individual exam. You may use your class notes, handouts posted on the course website, and James Munkres' *Topology*, but no other sources (animate or inanimate) should be used without first consulting your instructor.

This exam consists of **3 problems** with a total of 6 parts. You may handwrite or typeset your solutions but they are due by **10am on Friday, July 15, 2016**. Your solutions may be slid under my office door (Kemeny 219) or submitted electronically.

1. (10pts) Let $X = (-\infty, 0) \cup \{0', 0''\}$. Define a collection

$$\mathscr{B} = \{(a,b) \mid a < b < 0\} \cup \{(a,0) \cup \{0'\} \mid a < 0\} \cup \{(a,0) \cup \{0''\} \mid a < 0\}.$$

   Prove:

   (a) $\mathscr{B}$ is a basis for a topology on X.

   (b) $(X, \mathscr{T}_{\mathscr{B}})$ is not Hausdorff.

2. (15pts) Let $(X, \mathscr{T})$ be a topological space and $A, B \subset X$. Prove:

   (a) $\overline{A \cap B} \subset \overline{A} \cap \overline{B}$.

   (b) In general, $\overline{A} \cap \overline{B}$ is not a subset of $\overline{A \cap B}$. (Find a counterexample.)

   (c) $\overline{A \cup B} = \overline{A} \cup \overline{B}$.

3. (15pts) Let X be a topological space. Prove:

$$X \text{ is Hausdorff} \iff D = \{(x,x) \mid x \in X\} \text{ is closed in } X \times X.$$